Lecture 11

In this lecture we continue our study of matrices.

Nextweek's test:

— special functions
  - abs. value
  - floor & ceiling
  - factorials
  - exponentials
  - logs
  - polynomials

— summations & products

Exercises will go up soon.

More on the wonderful world of matrices

An $(n \times m)$-matrix can be multiplied together with an $(m \times k)$-matrix to give an $(n \times k)$-matrix. Then the first two matrices are said to be compatible for mult$^{\underline{n}}$, or to be "conformable".

<u>Eg.</u> $A = \begin{pmatrix} -2 & 0 & 4 & 1 \\ 1 & -3 & 0 & 2 \end{pmatrix}$

$$B = \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 2 & 0 \end{pmatrix}$$

A has order 2×4
B " " 4×3

$\therefore$ AB exists & has order 2×3

$$(2 \times \underbrace{4) \cdot (4}_{\text{same}} \times 3) \longrightarrow (2 \times 3)$$

$$\begin{pmatrix} -2 & 0 & 4 & 1 \\ 1 & -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 9 & -10 & 8 \\ & & \end{pmatrix}$$

$$\begin{array}{c} -2\cdot 1 \\ 0\cdot 0 \\ 4\cdot 3 \\ 1\cdot(-1) \\ \hline 9 \end{array} \qquad \begin{array}{c} -2\cdot 2 \\ 0\cdot 1 \\ 4\cdot(-2) \\ 1\cdot 2 \\ \hline -10 \end{array} \qquad \begin{array}{c} -2\cdot(-2) \\ 0\cdot 3 \\ 4\cdot 1 \\ 1\cdot 0 \\ \hline 8 \end{array}$$

$$\begin{pmatrix} -2 & 0 & 4 & 1 \\ 1 & -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 9 & -10 & 8 \\ -1 & 3 & -11 \end{pmatrix}$$

$$\begin{array}{c} 1\cdot 1 \\ -3\cdot 0 \\ 0\cdot 3 \\ 2\cdot(-1) \\ \hline -1 \end{array} \qquad \begin{array}{c} 1\cdot 2 \\ -3\cdot 1 \\ 0\cdot(-2) \\ 2\cdot 2 \\ \hline 3 \end{array} \qquad \begin{array}{c} 1\cdot(-2) \\ -3\cdot 3 \\ 0\cdot 1 \\ 2\cdot 0 \\ \hline -11 \end{array}$$

So $\quad AB = \begin{pmatrix} 9 & -10 & 8 \\ -1 & 3 & -11 \end{pmatrix}$.

Is this equal to BA?

Check: $BA = ?$

$(4 \times 3) \quad (2 \times 4)$

Not the same!

This mult$^{\underline{n}}$ cannot be done.
The matrices B and A (in <u>that</u> order) are not compatible for mult$^{\underline{n}}$.
So BA doesn't exist.

Do we always have $AB = BA$?
No — obviously not. (Maybe one exists and the other doesn't.)

Suppose they both exist. Must they be equal?

No.

A very easy counterexample:

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

Here's another counterexample:

$$\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$$

Here, AB and BA have <u>different order</u>

<u>Transpose</u>

From a matrix A we get its transpose A^T by <u>interchanging</u> the rows & columns.

We write down each row of A as a column of a new matrix, which becomes A^T.

E.g.

$$A = \begin{pmatrix} 2 & -3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \text{ — order } (2 \times 3)$$

$$A^T = \begin{pmatrix} 2 & -2 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} \text{ — order } (3 \times 2)$$

Notes

$$(A+B)^T = A^T + B^T$$

$$(\underbrace{A}_{(m\times n)}\underbrace{B}_{(n\times k)})^T = \underbrace{A^T}_{(n\times m)}\underbrace{B^T}_{(k\times n)} \quad ?$$

No — possibly AB exists but $A^T B^T$ doesn't.

$$\underbrace{(\underbrace{AB}_{m\times k})^T}_{k\times m} = \underbrace{\underbrace{B^T}_{(k\times n)}\underbrace{A^T}_{(n\times m)}}_{k\times m}$$